Prof. Dr. Alexander Pott

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG
FAKULTÄT FÜR MATHEMATIK

Final Exam, February 9, 2011
Mathematical Foundations
(2 hours)

(1) **(8 points)** Calculate the solutions of the following differential equation:

$$y' + \frac{y}{x} = x + \frac{2}{x} + 3,$$

where $y(-1) = 0$.

(2) **(9 points)** Calculate the solutions of the differential equation:

$$y'' + 5y' + 4 = 3 - 2x.$$

(3) **(12 points)** Consider the function

$$f(x, y) = xy + \frac{4}{x} + \frac{2}{y}, \quad x, y > 0.$$

(a) Compute the gradient and the Hessian matrix of f at (x_0, y_0).

(b) Calculate the stationary points of f, check them for local minimum or maximum points and compute the corresponding values of f.

(4) **(12 points)** Define matrices A and P as

$$A = \begin{pmatrix} -1 & 1 & 0 \\ -4 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}, P = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) Compute P^{-1}, $P \cdot A$, $A \cdot P$ and the transpose A^t of A.

(b) Compute the eigenvalues and the corresponding eigenvectors of A.

(5) (9 points) Let $a = \sqrt{2} + \sqrt{2}\mathbf{i}$ and $b = \frac{\sqrt{3}+\mathbf{i}}{4}$, where $\mathbf{i}$ is, as usual, a root of $f(x) = x^2 + 1$ in $\mathbb{C}$.

(a) Compute $a + b\mathbf{i}$ and $\frac{b}{a}$.

(b) Calculate $a^{2010}b^{2011}$ by using Euler expression.