Fakultät für Mathematik
Institut für Algebra und Geometrie
Prof. Dr. A. Pott

Mathematical Foundations, Exam A

13.02.2013

(1) **(4 pt.)** Find all solutions of the differential equation

$$y''(x) - 4y'(x) + 4y = 0.$$

(2) **(4 pt.)** Determine all eigenvalues and eigenvectors of

$$M = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}.$$

(3) **(5 pt.)** For which $a \in \mathbb{R}$ has the system of linear equations

$$\begin{array}{rcrcrcl} 2x & + & 4y & + & 2z & = & 2 \\ 2x & + & 3y & + & 3z & = & a \\ x & + & y & + & 2z & = & 1 \end{array}$$

no solution / exactly one solution / infinitely many solutions.

(4) **(4 pt.)** Compute

$$\int \frac{x^3}{x^4 - 1} \mathrm{d}x.$$

Note that this expression has basically the form $\int \frac{f'(x)}{f(x)} \mathrm{d}x$.

(5) **(5 pt.)** Compute the gradiant and the Hessian matrix (second partial derivatives) of

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y^2 e^x + y\cos(x) \\ y x^2 e^{y x^2} \end{pmatrix}.$$

(6) (a) **(2 pt.)** Compute $a, b \in \mathbb{R}$ such that

$$\frac{3\,\mathrm{i}}{\sqrt{2} - \mathrm{i}\sqrt{2}} = a + b \cdot \mathrm{i}.$$

(b) **(3 pt.)** Compute the Euler representation of

$$\omega = \frac{1}{2}(-1 + \mathrm{i}\sqrt{3}).$$

Use this to show $\omega^6 = 1$.

(7) **(5 pt.)** Solve the differential equation

$$y'(t) = 2\sqrt{y(t)}$$

with $y(0) = 3$.

(8) **(4 pt.)** Find the quadratic approximation of $\cos(x)$ at the point $x_0 = \pi$.

(9) **(4 pt.)** Compute

$$\det \begin{pmatrix} 0 & 2 & 1 & -2 \\ 1 & 3 & 2 & 0 \\ -1 & 3 & -1 & 2 \\ 1 & 2 & 1 & 0 \end{pmatrix}.$$

(10) **(5 pt.)** Find all stationary points of

$$f\begin{pmatrix} x \\ y \end{pmatrix} = e^{2x}(x + y^2 + 2y)$$

and determine whether f has a maxima or minima at these points.

(11) **(5 pt.)** Compute

$$\int \frac{1}{\cos^2(x)} \cdot x \; \mathrm{d}x.$$

You may use $\int \frac{1}{\cos^2(x)} \mathrm{d}x = \tan(x)$ and $\int \tan(x) \mathrm{d}x = \ln|\cos(x)|$.