Otto-von-Guericke-Universität Magdeburg
Institut für Algebra und Geometrie
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February 17, 2014

Mathematical Foundations - Exam

- First, please complete the form below.
- You have 120 minutes time for solving the problems.
- You have to give reasons/justifications for your solutions!
- At the end, please **write your name on all solutions** and **hand over this sheet, your solutions and the problem sheet!**
- You need 20 points for passing the exam. You may get up to 50 points for this class exercise and up to 3 points for your homeworks.

- - - - - - - - - - - - - - - - - to be filled in by the candidate - - - - - - - - - - - - - - - - -

Family name, first name:

Matrikel no.:

Program of study (INS, MSE, PNK etc.):

E-mail:

- - - - - - - - - - - - - - - - - to be filled in by the examiner - - - - - - - - - - - - - - - - -

Points from class exercise:

Points from homeworks:

Total of points:

Grade:

Date and signature:

(1) [4 points] Let

$$\mathbf{v} = (1, 0, -1) \quad \text{and} \quad \mathbf{w} = (2, -2, -1)\,.$$

Compute the lengths $|\mathbf{v}|$ and $|\mathbf{w}|$ as well as the size $\angle(\mathbf{v}, \mathbf{w})$ of the angle between $\mathbf{v}$ and $\mathbf{w}$.

(2) [6 points] Let

$$\mathbf{A} = \begin{pmatrix} -4 & -3 & -2 \\ -1 & 0 & 1 \\ 2 & 3 & 4 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 0 & 2 \\ 1 & 1 \\ 2 & 0 \end{pmatrix}.$$

Check if the following expressions exist and, if so, compute them:

$$\mathbf{A} \cdot \mathbf{B}, \qquad \mathbf{B} \cdot \mathbf{A}, \qquad \mathbf{B}^t, \qquad \det(\mathbf{A}).$$

(3) [5 points] For which $a \in \mathbb{R}$ does the linear equation system in the variables x_1, x_2, x_3,

$$\begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & a \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

have no solution / exactly one solution / infinitely many solutions?

(4) [5 points] Compute all eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 3 & 3 \\ 3 & -5 \end{pmatrix}.$$

(5) [4 points] Find all complex numbers that solve the equation

$$z^2 = 2\mathrm{i}$$

(where i is the imaginary unit). Give the solutions in Cartesian form.

(6) (a) [4 points] Compute the (definite) integral

$$\int_0^{\pi} x\cos(2x)\,dx\,.$$

(b) [4 points] Compute the indefinte integral

$$\int x\cos(x^2)\,dx\,.$$

(7) [6 points] Solve the initial value problem

$$y'(x) = 2x\cdot e^{-y(x)}\,, \qquad y(2) = 0\,.$$

(Don't forget to determine the largest interval where the solution is defined.)

(8) [5 points] Find *one particular* solution of the differential equation

$$y'(x) - 3y(x) = 10\sin(x)\,.$$

(You may use the method of guessing.)

(9) [7 points] Let

$$f(x,y) = x\sin(y) + \cos(x+y) \qquad ((x,y)\in\mathbb{R}^2).$$

Show that $(0,0)$ is a stationary point of f and check if $(0,0)$ is a local minimum point, a local maximum point or neither of that.