Mathematical Foundations (Winter 2014/2015)
Retake Exam

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(1) **(3 points)** Let

$$\mathbf{u} = (3, 5, -1) \quad \text{and} \quad \mathbf{v} = (4, -2, 2).$$

Compute the products $\mathbf{u}\mathbf{v}^T$ and $\mathbf{u}^T\mathbf{v}$, if they exist, and decide whether $\mathbf{u}$ and $\mathbf{v}$ are perpendicular.

(2) **(5 points)** Solve the system of linear equations

$$\begin{pmatrix} -1 & -5 & 3 \\ -2 & -7 & 0 \\ -1 & -4 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix}.$$

(3) **(4 points)** Fow which $h \in \mathbb{R}$ is the matrix

$$\begin{pmatrix} 2 & 1 & 3 & 3 \\ 1 & 5 & 3 & 0 \\ 2 & 4 & 7 & 0 \\ 1 & h & 2 & 0 \end{pmatrix}.$$

invertible?

(4) Let

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 4 & 1 \\ 0 & -2 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{u} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}.$$

(a) **(2 points)** Verify that $\mathbf{u}$ is an eigenvector of $\mathbf{A}$ by determining the corresponding eigenvalue.

(b) **(5 points)** Determine all other eigenvalues of $\mathbf{A}$.

(5) **(4 points)** Find the Euler expression of

$$e^{\mathbf{i}\pi/5}(\sqrt{2}e^{\mathbf{i}\pi/4} - e^{\mathbf{i}\pi/2}).$$

Please turn over.

(6) **(3 points)** Show that

$$\int \frac{1}{(\sin x)^2}\,dx = -\frac{1}{\tan x} + C,$$

where $C \in \mathbb{R}$ is an arbitrary constant.

(7) (a) **(4 points)** Compute the definite integral

$$\int_1^e x \ln(x)\,dx.$$

(Your answer may involve the Euler number e.)

(b) **(4 points)** Compute the indefinite integral

$$\int x\sqrt{1+x^2}\,dx.$$

(8) **(4 points)** Compute the general solution of

$$y'\,y\,e^{-x} = 1.$$

(9) **(6 points)** Solve

$$y'' + 2y' - 8y = 0$$

with the initial values $y(0) = 3$ and $y'(0) = 0$.

(10) **(6 points)** Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x, y) = e^x(x + y^2).$$

Determine the critical points of f and decide whether f has a local maximum, a local minimum, or neither of that at these points.