

Corrections for “Deformation of C^0 Riemannian metrics in the direction of their Ricci curvature”

1. (**Correction to Lemma 2.2 and Theorem 2.3**)

Lemma 2.2 und Thm. 2.3 are only correct under the assumption that

$$\begin{aligned} & \delta_1 h \leq g_0 \leq \delta_2 h, \\ \text{where } & |\delta_i - 1| \leq \varepsilon(n), i = 1, 2, \\ \text{where } & \varepsilon(n) \text{ is sufficiently small .} \end{aligned} \tag{0.1}$$

NOT for arbitrary $\delta_1, \delta_2 > 0$ (there are counter examples). In the rest of the paper I only consider δ_1, δ_2 as in (0.1). For this reason no problems occur.

Explanation:

In the proof of Lemma 2.2 the evolution equation I calculate for F is only correct under the assumption (0.1). NOT for arbitray δ_1, δ_2 . My F is simliar to the F considered by W.X. Shi in [2]. Shi calculates the evolution equation for such an F and I follow his method when calculating the evolution equation for my F . In equation (86) in Lemma 2.3 of [2] (page 240), Shi obtains an estimate of the form

$$LHS \leq \frac{m}{(1 - \delta)^3} (n^2 \sqrt{k_0} + 4 \tilde{\nabla}_k g_{ij} \tilde{\nabla}_k g_{ij}). \tag{0.2}$$

For $\delta_1 = 1$ and arbitrary $\delta_2 > 1$ this inequaluty is NOT necessarily valid for my F . I obtain an estimate of the form

$$LHS \leq c(n)m(1 + \delta_2)^m (n^2 \sqrt{k_0} + 4 \tilde{\nabla}_k g_{ij} \tilde{\nabla}_k g_{ij}).$$

This will only fulfill an estimate similar to (0.2) (and hence cause no problems in the rest of the derivation of the evolution equation for my F) if $(1 + \delta_2)^m$ is close enough to one. That is, if $\delta_2 > 0$ is small enough.

Theorem 2.3 uses Lemma 2.2.

2. (**Correction to Definition 6.4 in the non-compact case**)

In Defn. 6.4 (Lipschitz metrics on three manifolds with Ricci ≥ 0) I forgot to include the condition that (if M is non-compact) then the family of metrics $(^\alpha g)_{\alpha \in \mathbb{N}}$ appearing in Definition 6.4 satisfy

$$\sup_M |\text{Riem}(^\alpha g)| < \infty$$

for all α , which is used in the proof of Thm 1.4. This was corrected in [1] (below). (Defn.6.4 is only used in the application Thm 1.4).

3. (**Correction to Theorem 7.3 in the non-compact case**)

Thm 7.3 (a non-compact maximum principle) is not quite correct in the non-compact case. One needs to assume a bit more about the evolving metric if

M is non-compact: for example that the tensor N in question (appearing in the statement of Theorem 7.3) satisfies

$$N \geq -c_1 g \text{ on } M \times [0, T], \quad (0.3)$$

which is the case in all of the applications (Thms 1.2,1.3,1.4) of this paper (in [1] I showed that the condition (0.3) is satisfied for all tensors N which are considered in Thms 1.2,1.3,1.4.).

Bibliography

1. Simon, M. *Deforming Lipschitz metrics into smooth metrics while keeping their curvature operator non-negative*, Geometric Evolution Equations, Hsinchu Taiwan, 2002 (Conference proceedings), 167-179, Contemp.Math 367. Amer.Math Soc., Providenz,
2. Shi, Wan-Xiong., *Deforming the metric on complete Riemannian manifolds*. J.Differential Geometry, 30 (1989), 223–301.