Introduction to Algebraic Analysis

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Guiding questions

What is a **D-module**, why is it useful, and how to compute with it?

Content

The course treats some **algebraic aspects** of homogeneous, **linear partial differential equations** (PDEs) with polynomial coefficients, with an emphasis on computations. Algebraically, such PDEs are encoded as elements of the Weyl algebra, denoted D, which is a non-commutative $\mathbb{C}$-algebra gathering linear differential operators. Systems of PDEs correspond to left modules over D. These modules encode crucial information about their **analytic solutions**, such as their singularities, growth behavior, and the number of initial conditions that is required to uniquely encode the function.

The mathematical field of algebraic analysis contributed deep foundational insights to the theory of linear PDEs, but it is also useful in applications. Functions that can be encoded by an annihilating D-ideal are called **holonomic**, and these are ubiquitous in the sciences. Via **Gröbner basis techniques** in the Weyl algebra, one can modify D-ideals symbolically. This facilitates, for instance, a systematic representation of systems of PDEs in matrix form as well as the **computation** of series solutions to holonomic D-ideals which have regular singularities only.

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Format

The course is hands-on: the focus lies on the introduction of concepts from algebraic analysis and how to compute with them. The lectures are accompanied by tutorials, in which we discuss exercises and also employ computer algebra software.

Schedule

2 x 90 minutes per week. Ratio $^{\text{lectures}}/_{\text{tutorials}}$: $^3/_1$.

Prerequisites

Familiarity with. . .

  ◇ . . . basic concepts from commutative algebra.   ring, ideal, $\mathbb{C}$-algebra, modules
  ◇ . . . basic concepts from complex analysis is beneficial.   holomorphic fct.

Required concepts from algebraic geometry are going to be introduced en route.

Main reference (available via Springer Nature Link):

M. Saito, B. Sturmfels, and N. Takayama. *Gröbner deformations of hypergeometric differential equations*, volume 6 of *Algorithms and Computation in Mathematics*. Springer, 2000.

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