MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

CSC COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

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Model Reduction for Dynamical Systems

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What problems does this Lecture consider?

Any system of many Ordinary Differential Equations and/or Algebraic Equations

$$e_{11}\frac{dx_1}{dt}+\cdots e_{1n}\frac{dx_n}{dt}=a_{11}x_1+\cdots a_{an}x_n+b_1,$$
$$\vdots$$
$$e_{n1}\frac{dx_1}{dt}+\cdots e_{nn}\frac{dx_n}{dt}=a_{n1}x_1+\cdots a_{nn}x_n+b_n.$$

$$n\geq 10^4$$

$$\Longrightarrow \quad M\frac{d\boldsymbol{x}}{dt}=A\boldsymbol{x}+B$$

$$\boldsymbol{x}=(x_1,\ldots,x_n)$$

How to efficiently obtain the solution vector?

Examples

1D Inviscid Burgers' equation

$$\frac{\partial u(x,t;\mu)}{\partial t} + u(x,t;\mu)\frac{\partial u(x,t;\mu)}{\partial x} = 0,$$

$$x \in \Omega = [0,2]$$

$$t \in [0,T],$$

$$u(2,t;\mu) = u(0,t;\mu)$$

$$u(x,0;\mu) = \begin{cases} 1 + \frac{\mu}{2}\left(\sin\left(2\pi x - \frac{\pi}{2}\right) + 1\right) & \text{if } 0 \le x \le 1 \\ 1 & \text{otherwise} \end{cases}$$

Finite difference discretization in space domain ⇨

$$\frac{dU}{dt} = MU \odot U + BU$$

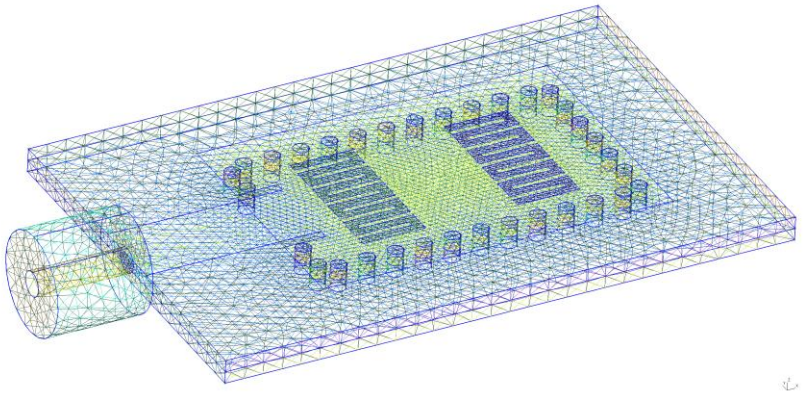
There are 1000 nonlinear ordinary differential equations. We want solutions corresponding to different initial conditions.

Examples

Substrate Integrated Waveguide Antenna

Time-harmonic Maxwell's equations

$$\nabla \cdot (\varepsilon \tilde{\mathbf{E}}) = \tilde{\rho}$$
$$\nabla \cdot (\mu \tilde{\mathbf{H}}) = 0$$
$$\nabla \times \tilde{\mathbf{E}} = -j\omega\mu \tilde{\mathbf{H}}$$
$$\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\varepsilon \tilde{\mathbf{E}}$$



Finite element discretization in space domain

$$(S + sU + s^2T)\boldsymbol{x} = sQ$$

There are 390,302 algebraic equations, $s = 2\pi j f_{EM}, f_{EM} \in [6, 8]GHz$
We need solution $\boldsymbol{x}$ at many values of f_{EM}.

Aim of model order reduction

Original model (discretized in space)

$$\Sigma \begin{cases} M(\mu)\dfrac{dx}{dt} = f(\mu, x) + Bu(t, \mu) \\ y(t, \mu) = Cx(t, \mu) \end{cases}$$

states $x \in \mathrm{R}^{n}$,

inputs $u(t) \in \mathrm{R}^{m}$

output $y(t) \in \mathrm{R}^{q}$

$$r << n$$

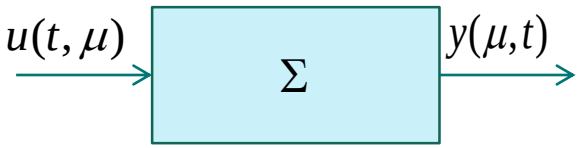
Reduced order model

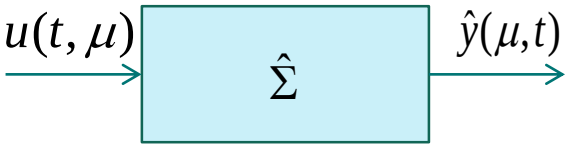
$$\hat{\Sigma} \begin{cases} \widehat{M}(\mu)\dfrac{dx}{dt} = \hat{f}(\mu, z) + \hat{B}u(t, \mu) \\ \hat{y}(t, \mu) = \hat{C}z(t, \mu) \end{cases}$$

states $z(t) \in \mathrm{R}^{r}$,

inputs $u(t) \in \mathrm{R}^{m}$

output $\hat{y}(t) \in \mathrm{R}^{q}$





$$\left|\left|y - \hat{y}\right|\right| < tol, \qquad \forall t, \mu$$

Some results

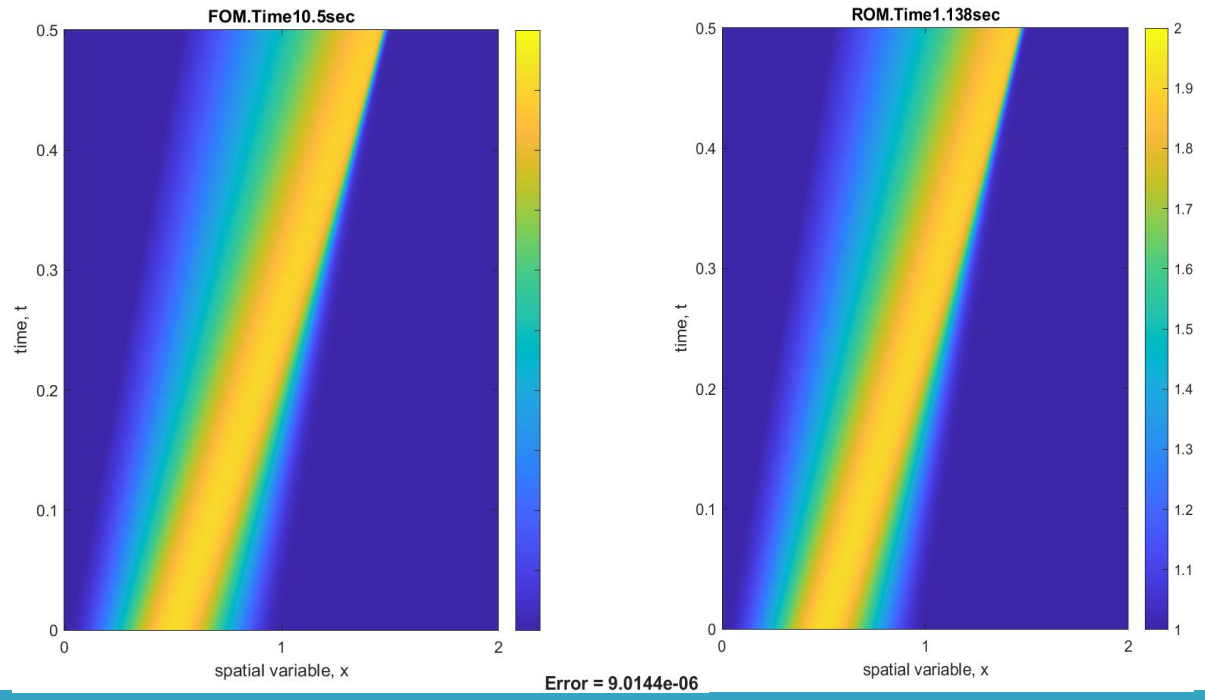
1D Inviscid Burgers' equation

Solving the original model: **10.5 seconds**

$$\frac{dU}{dt} = MU \odot U + BU$$

Solving the reduced model with 140 equations: **1.1 seconds**

$$\frac{d\hat{U}}{dt} = \hat{M}\hat{U} \odot \hat{U} + B\hat{U}$$



Some results

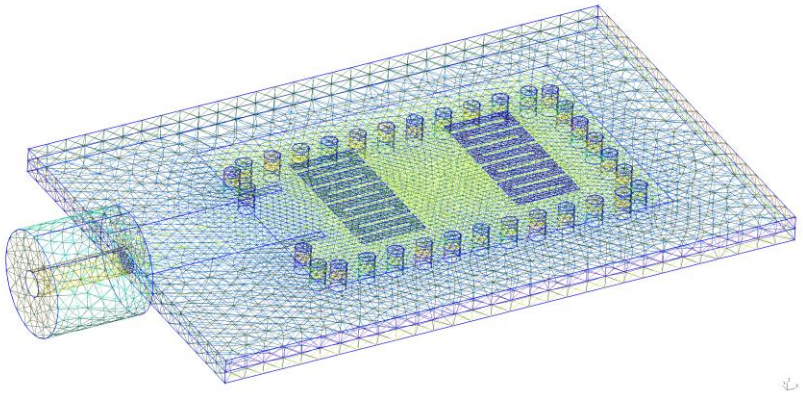
Substrate Integrated Waveguide Antenna

Solving the original model: **9273 seconds**

$$(S + sU + s^2T)\boldsymbol{x} = sQ$$

Solving the reduced model: **0.007 seconds**
Only 7 equations:

$$\left(\hat{S} + s\hat{U} + s^2\hat{T}\right)\hat{x} = s\hat{Q}$$



Some results

Shallow Water Equation

$$\partial_t h + \partial_x (hu) = 0,$$
$$\partial_t (hu) + \partial_x \left(hu^2 + \frac{1}{2} g h^2 \right) = -\frac{\nu}{\lambda} u,$$

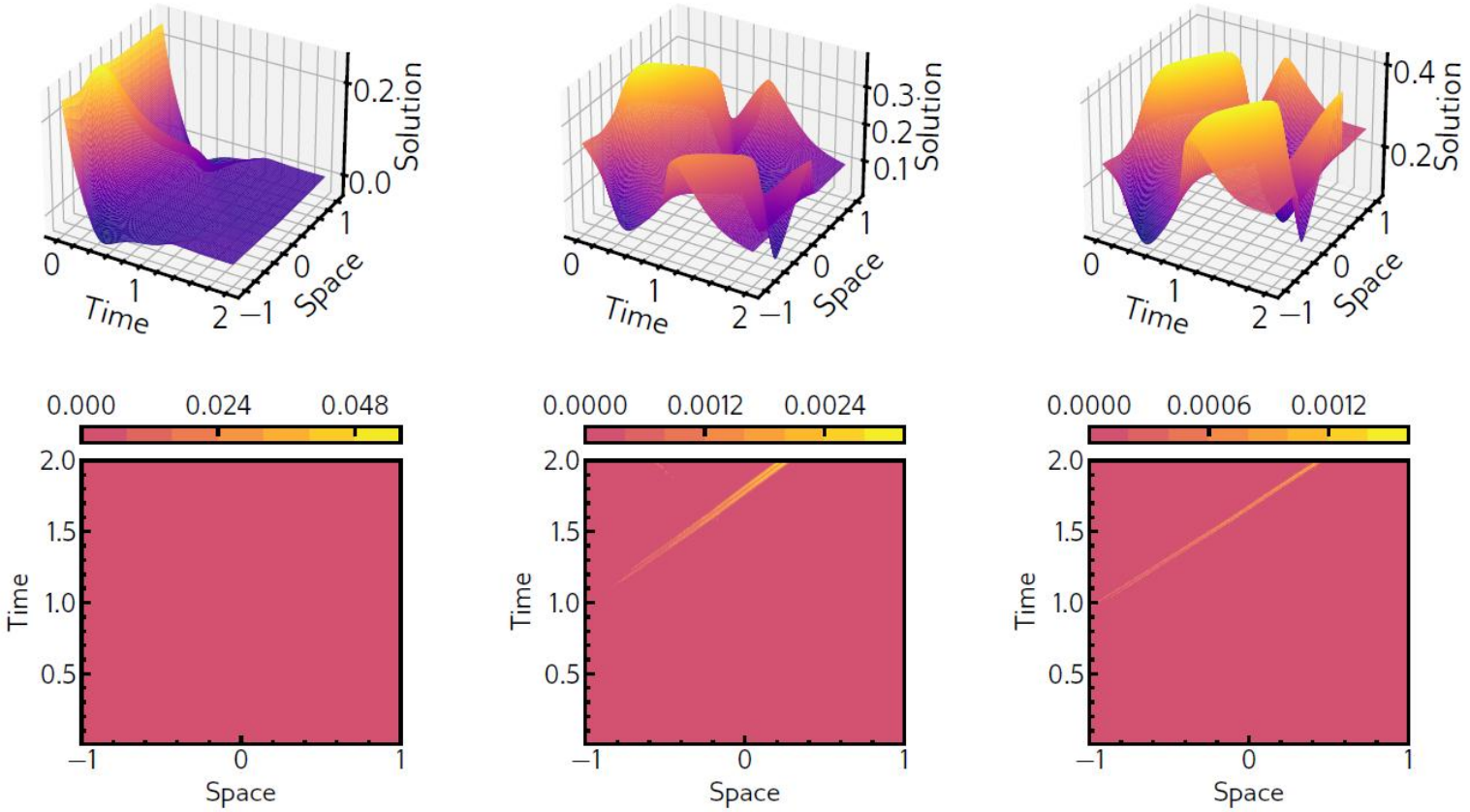
where $h(t, x)$ is the fluid height, $u(t, x)$ is the depth-averaged velocity, g is the gravitational acceleration, ν is the fluid's dynamic viscosity, and λ is the fluid's mean-free path. The initial fluid height and velocity are prescribed as follows:

$$h(0, x) = 1 + \exp(3 \cos(\pi(x + 0.5)) - 4),$$
$$u(0, x) = 0.25.$$

- The shallow water equations are typically used to model the behavior of water bodies like a lake or canal, as well as in weather forecasting

Some results

Shallow Water Equation



Original model solving time: **249 seconds**

Reduced model (in the form of neural network) solving time: **0.0122 seconds**

Conclusion

The question:
How to construct the ROMs for large-scale complex systems?

The lecture will provide many solutions.

Thank you for your attention!